

Supplementary Material for "Integrating point glacier mass balance observations into hydrologic model identification" : Cold content concept to delay melt water production

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1 Introduction - the cold content of a snowpack

The cold content of a snow pack is the energy required to bring the temperature of a dry snowpack to the temperature of melt $T_m = 0^\circ\text{C}$ (Marks et al., 1999). This is a useful concept to interpret the delay between air temperature raising above 0°C and actual melt outflow of a snowpack. In physical snowpack models that compute the snowmelt based on a complete energy balance (e.g. Essery and Etchevers, 2004; Marks et al., 1999), the cold content is either a state variable of the model or it can easily be computed from other state variables. Conceptual models that use a temperature-index approach to estimate snowmelt do not compute the energy balance of the snowpack and its thermal state can, therefore, not be quantified directly. Accordingly, there are very few examples in the literature where the two concepts (cold content, temperature-index approach) are juxtaposed. Thanks to the overview of seven conceptual snow modules of Valéry (2010), we are aware of the model CEQUeau (Morin, 2002) and Mordor (Garçon, 1996).

The Handbook of Hydrology (ASCE, 1996) proposes a simple equation to relate the cold content of the snowpack to the specific heat of ice c_i , the latent heat of fusion c_f , the average snow density ρ_s , the depth of the snowpack d_s and its average temperature deficit $T_m - T_s$:

$$w_c = \frac{c_i}{c_f} \frac{\rho_s}{\rho_w} d_s (T_m - T_s), \quad (1)$$

where ρ_w (kg m^{-3}) is the density of water and w_c is the cold content in mm of water equivalent (mm w.e.), i.e. w_c is the amount of water that has to be produced at the snow surface to release the required energy by freezing. This cold content has the effect of an initial loss that abstracts some of the melt energy.

2 Quantification of the cold content

In relative terms, i.e. compared to the depth of the snowpack h_s , this initial loss equals

$$\ell = \frac{w_c}{h_s} = \frac{c_i}{c_f} \frac{\rho_s}{\rho_w} (T_m - T_s). \quad (2)$$

The ratio $\frac{\rho_s}{\rho_w}$ equals between 0.1 and 0.5 for snow; furthermore, we have $c_i = 2.06 \text{ kJ kg}^{-1} \text{ } ^\circ\text{C}^{-1}$ at 0°C and $c_f = 334 \text{ kJ kg}^{-1}$, i.e. $\frac{c_i}{c_f} = 0.006$. It follows that this initial loss ℓ can only represent a few % of the total snow pack, which might, however, still represent an important loss relative to potential melt rates.

To quantify this loss relative to melt within a temperature-index modeling set-up, we first switch from the snowpack depth d_s in mm to the snowpack height in mm water equivalent, $h_s = d_s \frac{\rho_s}{\rho_w}$ (mm w.e.). Equation 1 becomes

$$w_c = \frac{c_i}{c_f} h_s (T_m - T_s). \quad (3)$$

To simplify, we omit hereafter $T_m = 0^\circ\text{C}$ and refer to $-T_s$ as the *snowpack temperature deficit*. The potential snowmelt M_{pot} for positive air temperatures, $T > 0^\circ\text{C}$, is computed as (see Eq. 1 of the paper)

$$M_{\text{pot}} = a_s T. \quad (4)$$

The ratio r_c of cold content to potential melt over one time step Δt becomes

$$r_c = \frac{w_c}{M_{\text{pot}} \Delta t} = \frac{c_i}{c_f} \frac{h_s}{a_s \Delta t} \frac{-T_s}{T}. \quad (5)$$

The amount of melted snow over Δt , h_m equals

$$h_m = \max[0, M_{\text{pot}} \Delta t - w_c]. \quad (6)$$

Combining the above two equations, we can compute the amount of melted snow relative to potential melt over Δt , $\frac{h_m}{M_{\text{pot}} \Delta t}$, as a function of the snow height h_s and of the ratio of air temperature and snowpack temperature deficit, $\frac{-T_s}{T}$ (where $-T_s > 0$ and $T > 0$) (see Fig. S1). For small ratios (high air temperature and snowpack temperature deficit close to 0°C) or small snow heights, h_m is close to the potentially meltable amount and the value decreases to zero

for high snow heights or high temperature ratios (high snowpack temperature deficit, low air temperature).

3 Using the cold content concept within a temperature-index snow-melt model

Trying to bring in some further energy balance components into the temperature-index approach might be seen as being in contradiction with the simplicity of the approach; it might even interfere with the reasons why the temperature-index approach actually works (see also Ohmura, 2001). We nevertheless made an attempt to include it in our simulations to explore any potential for improvement of our model given the data at hand.

We account for the effect of the cold content of the snowpack by estimating for each time step the temperature deficit of the simulated snowpack h_s . This temperature deficit is estimated assuming that it equals the average air temperature since the beginning of the building up of the snowpack. We then assume that this cold content abstracts "energy" (in terms of mm w.e.) from the potential melt during this time step, i.e. we obtain a time-variable initial abstraction of potential melt.

We implemented this into our existing snowpack model with the following algorithmic steps :

Append to the snowmelt computation at time step i the following steps :

- (1) If there is an existing snowpack, increment the snowpack age, Υ , by 1 time step, if there is no snowpack, set Υ to 0.
- (2) Update the snowpack temperature deficit assuming that it equals the average air temperature over a period corresponding to the snow age, T_Υ ; if $T_\Upsilon > 0^\circ\text{C}$, set $T_\Upsilon = 0^\circ\text{C}$.
- (3) Compute the cold content $w_c(i)$ for the available snow height $h_s(i)$, given this estimated temperature deficit.
- (4) If $T(i) > T_m$, compute the melt abstraction as

$$M_{\text{ab}}(i) = \min[M_{\text{pot}}(i), w_c(i)/\Delta t]. \quad (7)$$

- (5) Compute the effective melt

$$M(i) = \min[M_{\text{pot}}(i) - M_{\text{ab}}, h_s(i)]. \quad (8)$$

In analogy to the temperature-index approach for melting, the above approach assumes that average air temperature can be used as a "proxy" for the thermal state of the snowpack resulting from heat exchange processes with the

overlying atmosphere. This neglects heat transfer from rain, heat exchange with the underlying ground and due to phase changes (Lehning, 2005). And this does not explicitly solve the cold content balance.

The assumption that the snowpack temperature deficit corresponds to the average air temperature over the entire snow season, might, at a first glance, lead to an underestimation of this deficit for elevation bands with a permanent snowpack, since positive summer temperatures will keep the modeled temperature deficit of this snowpack close to zero (for the case study of this paper, the highest elevation band has an estimated mean annual temperature of around -2°C). However, field experience shows that Alpine firn at altitudes lower than 4000 m asl. is constantly close to melting, which is namely due to the high water content (Suter, 2002). We conclude that in absence of an energy balance, using cumulated air temperature as a proxy for the snowpack temperature deficit is a viable assumption for this type of Alpine catchments.

As can be anticipated based on Figure S1, introducing the thermal state into the model has a considerable effect on melt water production from high elevation bands with an important snow pack at the beginning of the melting season. With the previously calibrated melt parameters, the highest elevation bands do not release any melt, which has a 'dramatic' effect on the overall discharge simulation (see Fig. S2a). After re-calibration of all model parameters on discharge (minimizing the f_Q criterion described in the paper), the difference between the two model versions, albeit visible in a discharge plot, is hardly detectable in terms of overall model performance (see Fig. S2b). The new model parameters are given in Table S1 (all parameters are re-calibrated). It can be seen that to compensate for the melt abstraction due to the cold content, the summer melt parameter is increased and there is a negative accumulation correction with altitude.

4 Discussion

The presented approach to account for the cold content of the snowpack would need, of course, further testing. A question could be hereby, whether the snowpack temperature deficit could be better approximated by some temporally weighted average of the air temperature with snowfall (Garçon, 1996; Valéry, 2010).

Accounting for the cold content of the snowpack adds a degree-of-freedom that influences the discharge simulation as well as the glacier mass balance, contrary to the concept of snowpack retention capacity that only affects the discharge simulation. This new degree-of-freedom is obtained here without additional tunable parameter, which at least partly explains why this additional

degree-of-freedom does neither lead to a higher model performance for any of the reference data sets nor does it reduce the remaining trade-off between discharge simulation and glacier-wide annual mass balance.

The use of the concept of retention capacity to delay melt water production has to our view the advantage of being more straightforward and is not blurred by a pseudo-energy balance, which makes it very difficult to track whether the right results are obtained for the right reasons.

5 Conclusion

The presented analysis shows that it is difficult to show whether accounting for the cold content of snow in the context of a conceptual hydrological model without energy balance can contribute to improve the discharge simulation at a daily time step. In different simulation contexts and namely at much smaller time steps and spatial scales, including the cold content could have a perceptible effect on the pattern of melt water production. At such scales, the usefulness of the suggested approach is, however, questionable since its assumptions will break down at small time steps or spatial scales. Further research in this field could certainly contribute to more fully understand the usefulness of the cold content approach.

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Table S1

Calibrated parameter values of GSM-SOCONT as presented in the manuscript (see Table 1) and parameter values obtained for the new model structure accounting for the thermal state (TS) of the snowpack.

Name	Unit	Best f_Q without TS	Best f_Q with TS
a_i	$\text{mm d}^{-1} \text{ }^\circ\text{C}^{-1}$	6.3	5.4
a_{sw}	$\text{mm d}^{-1} \text{ }^\circ\text{C}^{-1}$	5.8	5.7
a_{ss}	$\text{mm d}^{-1} \text{ }^\circ\text{C}^{-1}$	3.1	5.2
k_i	d^{-1}	3.5	5.7
k_s	d^{-1}	5.9	6.2
A	mm	1038	4589
$\log(k)$	$\log(\text{s}^{-1})$	-7.8	-10.8
β	$\text{m}^{4/3} \text{ s}^{-1}$	20 232	28 765
η_s	-	0.38	0.01
ρ_w	$^\circ\text{C m}^{-1}$	-0.0013	-0.0004
ρ_s	$^\circ\text{C m}^{-1}$	-0.0041	-0.0032
γ_r	$\% (100 \text{ m})^{-1}$	-2.5	-5.7
γ_w	$\% (100 \text{ m})^{-1}$	7.5	12.0
γ_s	$\% (100 \text{ m})^{-1}$	6.4	-6.6

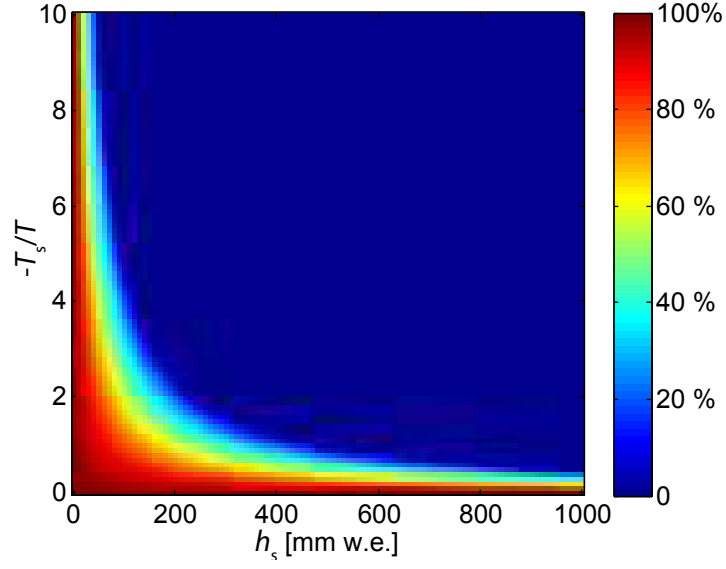


Figure S1. Melted snow h_m (mm w.e.) relative to potential melt M_{pot} (mm w.e. d^{-1}) over time step Δt as a function of the ratio $\frac{-T_s}{T}$ (snow temperature deficit / air temperature) and of the snow height h_s ; season: summer, $a_s = 3.1$ ($\text{mm d}^{-1} \text{ } ^\circ\text{C}^{-1}$)

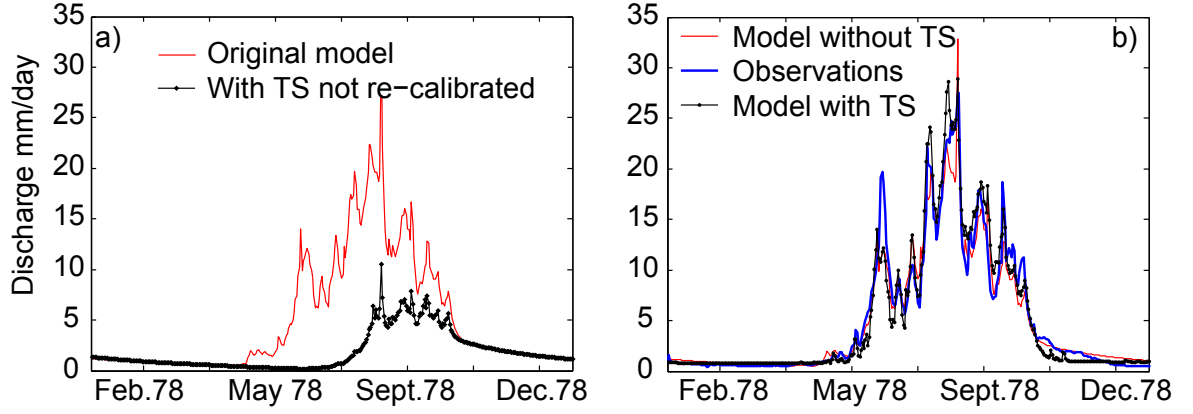


Figure S2. Simulated discharge; left: effect of adding thermal state (TS) module to the precipitation-discharge model; right: observed discharge and simulations corresponding to the final calibrated model of the manuscript (not accounting for thermal state) and the calibrated model with thermal state.